kd education academy (9582701166

Time : 10 Hour | STD 11 Maths | Total Marks : 110

kd90+ ch- 5 linear inequalities

*** Choose the right answer from the given options. [1 Marks Each]** **[40]**

1. Ordered pair that satisfy the equation x + y + 1 < 0 is:

   (A) (0, -1) (B) (-2,0) (C) (2, -4) (D) Both (B) and (C)

Ans. :

d. Both (B) and (C)

Solution:

Given inequation is x + y + 1 < 0
From option A, 0 + (-1) + 1 < 0
⇒ 0 < 0 which is false
Hence, (0, -1) is not a solution.
From option B, -2 + 0 + 1 < 0
⇒-1 < 0 which is true
Hence, (-2,0) there is a solution.
From option C, 2 - 4 + 1 < 0
⇒ -1 < 0 which is true.
Hence, (2,-4) is a solution.

2. If the cube roots of unity are $1,\omega$ and ω^2, then the roots of the equation $(\mathrm{x}-1)3+8=0,$ are:

   (A) $-1,1+2\omega,1+2\omega^2$ (B) $-1,1+2\omega,1-2\omega^2$

   (C) $-1,-1,-1$ (D) $-1,-1+2\omega-1-2\omega^2$

Ans. :

b. $-1,1+2\omega,1-2\omega^2$

3. If α and β are the roots of the equation $\mathrm{x}^2-\mathrm{x}+1=0,$ then a $2009+\beta^{2009}$ is equal to:

   (A) -2 (B) -1 (C) 1 (D) 2

Ans. :

c. 1

4. The value of x for which $|\mathrm{x}+1|+\sqrt{(\mathrm{x}-1)}=0$

   (A) 0 (B) 1 (C) -1 (D) No value of x

Ans. :

d. No value of x

Solution:

Given, $|\mathrm{x}+1|+\sqrt{(\mathrm{x}-1)}=0,$ where each term is non - negative.

So, $|x+1|=0$ and $\sqrt{(x\text{-}1)}=0$ should be zero simultaneously.
i.e. $x=-1$ and $x=1$, which is not possible.
So, there is no value of x for which each term is zero simultaneously.

5. The longest side of a triangle is 2 times the shortest side and the third side is 4cm shorter than the longest side.If the perimeter of the triangle is at least 61cm, find the minimum length of the shortest side.

(A) 7 (B) 9 (C) 11 (D) 13

Ans. :

d. 13

Solution:

Let shortest side be x.Then longest side $=2x$.
Third side $=2x-4$.
Given, perimeter of triangle is at least 61cm
$\Rightarrow x+2x+2x-4 \geq 61 \Rightarrow 5x \geq 65 = x \geq 13$.
Minimum length of the shortest side is 13cm.

6. If (1 – p) is a root of quadratic equation x^2 + px + (1-p) = 0, then its roots are:

(A) 0, 1 (B) – 1, 1 (C) 0, -1 (D) – 1, 2

Ans. :

c. 0, -1

7. The solution of the inequality $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$ is:

(A) $x \in (2, \infty)$ (B) $x \in [-2, \infty)$ (C) $x \in [\infty, 2)$ (D) $x \in [2, \infty)$

Ans. :

d. $x \in [2, \infty)$

Solution:

Given, $\frac{3(x-2)}{5} \geq \frac{5(2-x)}{3}$
$\Rightarrow 3(x–2) \times 3 \geq 5(2–x) \times 5$
$\Rightarrow 9(x–2) \geq 25(2–x)$
$\Rightarrow 9x–18 \geq 50–25x$
$\Rightarrow 9x–18+25x \geq 50$
$\Rightarrow 34x-18 \geq 50$
$\Rightarrow 34x \geq 50+18$
$\Rightarrow 34 \geq 68$
$\Rightarrow x \geq \frac{68}{34}$
$\Rightarrow x \geq 2$
$\Rightarrow x \in [2, \infty)$

8. If $|x+3| \geq 10$, then:

(A) $x \in (-13, 7]$ (B) $x \in (-13, 7)$

(C) $x \in (-\infty, -13] \cup [7, \infty)$ (D) $x \in (-\infty, -13) \cup [7, \infty)$

Ans. :

c. $x \in (-\infty, -13] \cup [7, \infty)$

9. If x is a natural number and $20x \leq 100$ then find solution set of x.

(A) {0, 1, 2, 3, 4, 5} (B) {1, 2, 3, 4, 5} (C) {1, 2, 3, 4} (D) {0, 1, 2, 3, 4}

Ans. :

b. {1, 2, 3, 4, 5}

Solution:

$20x \leq 100$

Dividing by 20 on both sides, $x \leq \frac{100}{20} \Rightarrow x \leq 5$

Since x is a natural number so x = 1, 2, 3, 4, 5.

10. If |x−1| x - 1 > 5, then:

(A) $x \in (-4, 6)$ (B) $x \in [-4, 6]$

(C) $x \in (-\infty, -4) \cup (6, \infty)$ (D) $x \in (-\infty, -4) \cup [6, \infty)$

Ans. :

c. $x \in (-\infty, -4) \cup (6, \infty)$

Solution:

|x−1| > 5

⇒ x − 1 > 5 or x − 1 < −5

⇒ x > 5 + 1 or x < −5 + 1

⇒ x > 6 or x < −4

$\Rightarrow x \in (-\infty, -4) \cup (6, \infty)$

11. If $4x + 3 < 6x + 7$, then $x \in$

(A) $(2, \infty)$ (B) $(-2, \infty)$ (C) $(-\infty, 2)$ (D) $(-\infty, \infty)$

Ans. :

b. $(-2, \infty)$

12. A solution is to be kept between 77° F and 86° F.What is the range in temperature in degree Celsius (C) if the $\frac{\text{Celsius}}{\text{Fahrenheit}}$(F) conversion formula is given by $F = \frac{9}{5c} + 32^\circ$

(A) $[15^\circ, 20^\circ]$ (B) $[20^\circ, 25^\circ]$ (C) $[25^\circ, 30^\circ]$ (D) $[30^\circ, 35^\circ]$

Ans. :

c. $[25^\circ, 30^\circ]$

Solution:

$F = \frac{9}{5c} + 32^\circ$

$C = F - 32^\circ \times \frac{5}{9}$

$77^\circ \leq F \leq 86^\circ$

$\Rightarrow 77^\circ - 32^\circ \leq F - 32^\circ \leq 86^\circ - 32^\circ$

$\Rightarrow 45° \leq F - 32° \leq 54°$

$\Rightarrow 45° \times \frac{5}{9} \leq (F - 32°) \times \frac{5}{9} \leq 54° \times \frac{5}{9}$

$\Rightarrow 25° \leq C \leq 30°$

13. Solution of $|3x + 2| < 1$ is:

(A) $\left[-1, \frac{-1}{3}\right]$ (B) $\left(\frac{-1}{3}, -1\right)$ (C) $\left(-1, \frac{-1}{3}\right)$ (D) None of these

Ans. :

c. $\left(-1, \frac{-1}{3}\right)$

14. The quadratic equations $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common. The other roots of the first and second equations are integers in the ratio 4 : 3.Then, the common root is:

(A) 2 (B) 1 (C) 4 (D) 3

Ans. :

a. 2

15. If x is a whole number and $10x \leq 50$ then find solution set of x.

(A) {0,1, 2, 3, 4, 5} (B) {1, 2, 3, 4, 5} (C) {1, 2, 3, 4} (D) {0, 1, 2, 3, 4}

Ans. :

a. {0,1, 2, 3, 4, 5}

Solution:

$10x \leq 50$

Dividing by 10 on both sides, $x \leq \left(\frac{50}{10}\right) \Longrightarrow x \leq 5$

Since x is a whole number so x = 0, 1, 2, 3, 4, 5.

16. The length of a rectangle is three times the breadth.If the minimum perimeter of the rectangle is 160cm, then:

(A) breadth > 20cm (B) length < 20cm (C) breadth x ≥ 20cm (D) length ≤ 20cm

Ans. :

c. breadth x ≥ 20cm

Solution:

Let x be the breadth of a rectangle.

So, length = 3x

Given that the minimum perimeter of a rectangle is 160cm.

Thus, $2(3x + x) \geq 160$

$\Rightarrow 4x \geq 80$

$\Rightarrow x \geq 20$

17. Write the solution of inequality $\frac{1}{5}\left(\frac{3x}{5} + 4\right) \geq \frac{1}{3}(x - 6)$.

(A) $x \leq \frac{105}{8}$ (B) $x \geq \frac{105}{8}$ (C) $x \geq 120$ (D) $x \leq 120$

Ans. :

a. $x \le \frac{105}{8}$

Solution:

$\frac{1}{5}\left(\frac{3x}{5}+4\right) \ge \frac{1}{3}(x-6).$

$\Rightarrow 3\left(\frac{3x}{5}+4\right) \ge 5\left(x-6\right)$

$\Rightarrow \left(\frac{9x}{5}+12\right) \ge 5x-6$

$\Rightarrow (30+12) \ge -\frac{9x}{5}+5x$

$\Rightarrow 42 \ge \frac{-9x+25x}{5}$

$\Rightarrow 42 \ge \frac{16x}{5}$

$\Rightarrow \frac{42\times5}{16} \ge x$

$x \le \frac{105}{8}$

Therefore option (1) is the correct answere.

18. The cost and revenue functions of a product are given by C(x) = 20x + 4000 and R(x) = 60x + 2000, respectively, where x is the number of items produced and sold. How many items must be sold to realise some profit?

(A) Less than 40 (B) More than 50 (C) Less than 50 (D) Exactly 50

Ans. :

b. More than 50

19. The sum of four numbers in AP is 20.The numbers are such that the ratio of the product of first and fourth is to the product of second and third as 2 : 3.The greatest number is:

(A) 8 (B) 7 (C) 14 (D) 4

Ans. :

a. 8

Solution:

Let the four terms be a − 3d, a − d, a + d, and a + 3d with common difference 2d.sum = 4a = 20a = 5a = 5

$\frac{\text{(a-3d)(a+3d)}}{\text{(a-d)(a+d)}} = \frac{2}{3}$

$\frac{25-9d^2}{25-d^2} = \frac{2}{3}$

$d^2 = 1 \Rightarrow d = 1$

Largest term = a + 3d = 5 + 3(1) = 8

20. Rahul obtained 20 and 25 marks in first two tests.Find the minimum marks he should get in the third test to have an average of at least 30 marks.

(A) 60 (B) 35 (C) 180 (D) 45

Ans. :

d. 45

Solution:

Average is at least 30 marks.

Let x be the marks in 3^{rd} test.

Average $= \frac{(20+25+x)}{3} \geq 30$

$\Rightarrow 45 + x \geq 90 \Rightarrow x \geq 90 - 45 \Rightarrow x \geq 45.$

Minimum marks in 3^{rd} test should be 45.

21. If $7x + 3 < 5x + 9$ then $x \in$

(A) $(-\infty, 3]$ (B) $(-\infty, \infty)$ (C) $(-\infty, 3)$ (D) $[3, \infty)$

Ans. :

c. $(-\infty, 3)$

22. If Ram has x rupees and he pay 40 rupees to shopkeeper then find range of x if amount of money left with Ram is at least 10 rupees is given by inequation, ___?

(A) x ≥ 10 (B) x ≤ 10 (C) x ≤ 50 (D) x ≥ 50

Ans. :

d. x ≥ 50

Solution:

Amount left is at least 10 rupees i.e.amount left ≥ 10.

x - 40 ≥ 10 ⇒ x ≥ 50.

23. A pack of coffee powder contains a mixture of x gms of coffee and y gms of choco. The amount of coffee powder is greater than that of chocolate and each pack weights at least 10g. Which of the followinginequalities describe the given condition?

(A) $x < y$ (B) $x + y \geq 10$ (C) $x + y \leq 10$ (D) $x > y$

Ans. :

b. $x + y \geq 10$

Solution:

The coffee powder is greater than choco.

hence, $x > y$

each pack is at least 10gm.

24. The value of a for which one root of the quadratic equation $(a^2 - 5a + 3)x^2 + 3a - 1)x + 2$ is twice as large as the other, is:

(A) $\frac{2}{3}$ (B) $\frac{-2}{3}$ (C) $\frac{1}{3}$ (D) $\frac{-1}{3}$

Ans. :

a. $\frac{2}{3}$

25. Find all pairs of consecutive odd natural numbers, both of which are larger than 10, such that their sum is less than 40.

(A) (11, 13), (13, 15), (15, 17), (17, 19) (B) (11, 13), (13, 15), (15, 17)

(C) (21, 23), (23, 25), (25, 27), (27, 29) (D) (15, 17), (17, 19), (19, 21), (21, 23)

Ans. :

a. (11, 13), (13, 15), (15, 17), (17, 19)

26. The number of pairs (a, b) of positive real numbers satisfying $a^4 + b^4 < 1$ and $a^2 + b^2 > 1$ is

(A) 0 (B) 1 (C) 2 (D) More than 2

Ans. : d

(d)

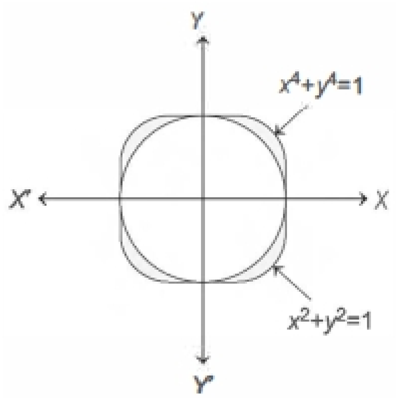
We have,

$a^4 + b^4 < 1$ and $a^2 + b^2 > 1$

The graph of $x^2 + y^2 = 1$ and $x^4 + y^4 = 1$ are

Clearly from graph.

There are many positive real number (a, b) satisfying $a^4 + b^4 < 1$ and $a^2 + b^2 > 1$



27. In a cinema hall, the charge per person is ₹200. On the first day, only 60% of the seats were filled. The owner decided to reduce the price by 20% and there was an increase of 50% in the number of spectators on the next day. The percentage increase in the revenue on the second day was

(A) 50 (B) 40 (C) 30 (D) 20

Ans. : d

(d)

Let total seats $= x$

Ticket price of each seat $= 200\, Rupees$

On first day 60% of seats over filled

$\because$ Total revenue $= \frac{60}{100} x \times 200 = 120x$

On second day

Ticket price $= 200 - 20\%$ of $200 = 160$

Total seat filled on 2nd day

$= \frac{60}{100} x + \frac{50}{100} \times \frac{60x}{100} = \frac{90x}{100}$

Total revenue on 2^{nd} day

$= \frac{90x}{100} \times 160 = 144x$

Percentage increase in revenue on 2^{nd} day

$= \left(\frac{144x - 120x}{120x}\right) \times 100$

$= \frac{24}{120} \times 100 = 20\%$

28. There are three kinds of liquids X, Y, Z. Three jars J_1, J_2, J_3 contains $100ml$ of liquids X, Y, Z respectively. By an operation we mean three steps in the following order

- stir the liquid in J_1 and transfer $10\,ml$ from J_1 into J_2
- stir the liquid in J_2 and transfer $10\,ml$ from J_2 into J_3
- stir the liquid in J_3 and transfer $10\,ml$ from J_3 into J_1.

After performing the operation four times, let x, y, z be the amounts of X, Y, Z respectively, in J_1. Then,

(A) $x > y > z$ (B) $x > z > y$ (C) $y > x > z$ (D) $z > x > y$

Ans. : b

(b)

We have, three kind of liquids x, y, z and three jars J_1, J_2, J_3 contains $100ml$ of liquids X, Y, Z respectively.

When $10\,ml$ of J_1 transfer to J_2 $\therefore J_1 = 90ml$ of $X, J_2 = 100\,ml$ of Y and $10\,ml$ of X.

When $10\,ml$ of J_2 transfer to J_3 $J_2 = \frac{1000}{11}$ of Y and $\frac{100}{11}$ of $X, J_3 = 100\,ml$ of $Z_1, \frac{100}{11}$ of Y and $\frac{10}{11}$ of X

When $10\,ml$ of J_3 transfer to J_1 $J_3 = \frac{1100}{11}$ of $Z_1, \frac{1100}{11}$ of $Y_1, \frac{110}{11}$ of X and $J_1 = 90 + 10 \times \left(\frac{1}{11}\right)^2$ of $X_1, \frac{100}{121}$ of Y and $\frac{100}{11}$ of Z

Similarly, we can find four operation of amount of X, Y, Z in J_1

We get $x > z > y$

29. Suppose the height of a pyramid with a square base is decreased by $p\%$ and the lengths of the sides of its square base are increased by $p\%$ (where, $p > 0$). If the volume remains the same, then

(A) $50 < p < 55$ (B) $55 < p < 60$ (C) $60 < p < 65$ (D) $65 < p < 70$

Ans. : c

(c)

Let the side of square base of pyramid is xm and height of pyramid is $y\,m$

Volume of pyramid $= \frac{1}{3}$ area of base $\times$ height $= \frac{1}{3}x^2y$

When x is increased by $p\%$ then new length $= x + p\%$ of $x = \left(\frac{100+p}{100}\right)x$

When y is decreased by $p\%$, then new height

$= y - p\% \text{ of } y = \left(\frac{100-p}{100}\right)y$

Now, volume is same.

$\therefore \frac{1}{3}x^2y = \frac{1}{3}\left(\frac{100+p}{100}x\right)^2\left(\frac{100-p}{100}\right)y$

$\Rightarrow \quad 1 = \left(\frac{100+p}{100}\right)^2\left(\frac{100-p}{100}\right)$

$\Rightarrow (100)^2(100) = \left(10000+200p+p^2\right)(100-p)$

$\Rightarrow p^2+100p-100^2=0$

$\Rightarrow p^2+100p+(50)^2=(100)^2+(50)^2$

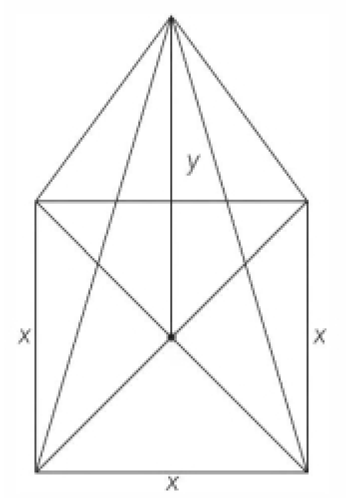
$\Rightarrow \quad (p+50)^2 = 12500$

$\Rightarrow \quad p+50=\sqrt{12500}=11180$

$\Rightarrow \quad p = 11180-50$

$\Rightarrow \quad p = 6180$

$60 < p < 65$



30. The solution set of $|x-1| \le -1$ is...

(A) $(0,2)$ (B) $[0,2]$ (C) $(-\infty,-1]\cup[1,\infty)$ (D) $\emptyset$

Ans. : (D) $\emptyset$

31. The number of ordered pairs (x,y) of integers satisfying $x^3+y^3=65$ is

(A) 0 (B) 2 (C) 4 (D) 6

Ans. : b

(b)

$x^3+y^3=65$

Let $x,y>0$

Clearly, $(1,4)$ and $(4,1)$ holds for x or $y \ge 6$ difference of two cubes is always greater than or equal to 91 . Hence, only 2 ordered pair possible.

32. $\frac{|x-1|}{x-1} \le 0$ then $x \in$

(A) $(-\infty,1)$ (B) $(1,\infty)$ (C) $(-1,1)$ (D) ϕ

Ans.: (A) $(-\infty,1)$

33. If $|x-2| \ge |x-4|$ then $x \in \ldots$

(A) $[2,4]$ (B) $[3,\infty)$ (C) $[3,6]$ (D) $[-4,-2]$

Ans. : (B) $[3,\infty)$

34. $\left|x+\frac{1}{x}\right| \geq 2$ then $x \in$

(A) $\mathrm{R}-\{0\}$ (B) $\mathrm{R}-\{\pm 1\}$ (C) R (D) 0

Ans.: (A) $\mathrm{R}-\{0\}$

35. The solution set of $x<5$ and $x \geq 2$ is...

(A) $(2,5)$ (B) $[2,5)$ (C) $(2,5]$ (D) $[2,5]$

Ans. : (B) $[2,5)$

36. The solution set of $\frac{x^2}{x^2+1}<0$ is

(A) 0 (B) $(-1,1)$ (C) ϕ (D) R

Ans. : (C) ϕ

37. If $|x-2| \geq 8$ then $x \in$

(A) $(-6,10)$ (B) $(-\infty,-6) \cup(10, \infty)$

(C) $(-\infty,-6) \cup(10, \infty)$ (D) $(-\infty,-6] \cup[10, \infty)$

Ans. : (D) $(-\infty,-6] \cup[10, \infty)$

38. The solution set of $x^2 \leq 9$ is

(A) $[-3,3]$ (B) $(-3,3)$

(C) $(-\infty,-3) \cup(3, \infty)$ (D) ϕ

Ans.: (A) $[-3,3]$

39. The solution set of $x^2 \leq 4$ is......

(A) $[-2,2]$ (B) $(-2,2)$ (C) $(-\infty,-2] \cup[2, \infty)$ (D) $\emptyset$

Ans.: (A) $[-2,2]$

40. The number of ordered pairs (a, b) of positive integers such that $\frac{2a-1}{b}$ and $\frac{2b-1}{a}$ are both integers is

(A) 1 (B) 2 (C) 3 (D) more than 3

Ans. : c

(c)

For positive integers ' a ' and ' b ' the numbers $\frac{2a-1}{b}$ and $\frac{2b-1}{a}$ are integers if ' a ' and ' b ' and odd integers because ' $2a-1$ ' and ' $2b-1$ ' are odd integers.

Now, let $\frac{2a-1}{b}=\alpha$ and $\frac{2b-1}{a}=\beta$, where α, β are integers. then $2a-1=\alpha b$ and $2b-1=\beta a$ so $4a-2=\alpha(\beta a+1)$

$a=\frac{\alpha+2}{4-\alpha\beta}$

$\because a$ is an integer, then $0<\alpha\beta<4$

$\therefore$ Possible value of $\alpha=1,2$ or 3 and $\beta=1,2$ or 3 such that $0 < \alpha\beta < 4$

Now, when $(\alpha, \beta) = (1,1)$, then $(a, b) = (1,1)$

When $(\alpha, \beta) = (1,2)$, then (a, b) have no value

When $(\alpha, \beta) = (1,3)$, then $(a, b) = (3,5)$ and similarly when $(\alpha, \beta) = (3,1)$, then $(a, b) = (5,3)$

So, number of ordered pairs (a, b) is 3.

*** Given section consists of questions of 2 marks each.** **[6]**

41. Solve the inequality $\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$ for real x.

Ans. : Here $\frac{(2x-1)}{3} \geqslant \frac{(3x-2)}{4} - \frac{(2-x)}{5}$

$\Rightarrow \frac{2x}{3} - \frac{1}{3} \geqslant \frac{3x}{4} - \frac{2}{4} - \frac{2}{5} + \frac{x}{5}$

$\Rightarrow \frac{2x}{3} - \frac{3x}{4} - \frac{x}{5} \geqslant \frac{-2}{4} - \frac{2}{5} + \frac{1}{3}$

$\Rightarrow \frac{40x-45x-12x}{60} \geqslant \frac{-30-24+20}{60}$

$\Rightarrow \frac{-17x}{60} \geqslant \frac{-34}{60}$

Multiplying both sides by 60, we have

$-17x \geqslant -34$

Dividing both sides by -17, we have

$\frac{-17x}{-17} \leqslant \frac{-34}{-17}$

$\Rightarrow x \leqslant 2$

Thus the solution set is $(-\infty, 2]$

42. The marks obtained by a student of Class XI in first and second terminal examinations are 62 and 48, respectively. Find the minimum marks he should get in the annual examination to have an average of at least 60 marks.

Ans. : Let x be the marks obtained by student in the annual examination.

Average = $\frac{62+48+x}{3} \geq 60$

$\Rightarrow \frac{62+48+x}{3} \geq 60$

$\Rightarrow$ 110 + x $\geq$ 60

$\Rightarrow$ x $\geq$ 70

Thus, the student must obtain a minimum of 70 marks to get an average of at least 60 marks.

43. Find all pairs of consecutive odd natural number, both of which are larger than 10, such that their sum is less than 40.

Ans. : Let x be the smaller of the two consecutive odd natural numbers. Then the other odd integer is x+2.

It is given that both the natural number are greater than 10 and their sum is less than 40.

$\therefore$ x > 10 and, x + x + 2 < 40

$\Rightarrow$ x > 10 and 2x < 38

⇒ x > 10 and x <19

⇒ 10 < x < 19

⇒ x = 11,13,15,17   [∵ x is an odd number]

Hence, the required pairs of odd natural number are (11, 13), (13, 15), (15, 17) and (17, 19).

*** Given section consists of questions of 3 marks each.** **[36]**

44. A manufacturer has 600 litres of a 12% solution of acid. How many litres of a 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18%?

Ans. : Let x liters of 30% acid solution is required to be added to the mixture.

Then Total mixture = (x + 600) litres

Therefore, 30% x + 12% of 600 > 15% of (x + 600)

& 30% x + 12% of 600 < 18% of (x + 600)

⇒ $\frac{30x}{100}+\frac{12}{100}(600)>\frac{15}{100}$ (x + 600) and $\frac{30x}{100}+\frac{12}{100}(600)<\frac{18}{100}$ (x + 600)

⇒30x + 7200 > 15x + 9000 and 30x + 7200 < 18x + 10800

⇒ 15x > 1800 and 12x < 3600

⇒ x > 120 and x < 300,

i.e. 120 < x < 300

Thus, the number of litres of the 30% solution of acid will have to be more than 120 litres but less than 300 litres.

45. Solve the inequality and show the graph for the solution on number line: 5x – 3 $\geq$ 3x – 5

Ans. : Here $5x-3\geqslant 3x-5$

⇒ $5x-3x\geqslant -5+3$

⇒ $2x\geqslant -2$

Dividing both sides by 2, we have

$x\geqslant -1$

The solution set is $[-1,\infty)$

The representation of the solution set on the number line is

46. Solve the inequality and show the graph for the solution on number line: $\frac{x}{2}\geq\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

Ans. : Here $\frac{x}{2}\geqslant\frac{(5x-2)}{3}-\frac{(7x-3)}{5}$

⇒ $\frac{x}{2}\geqslant\frac{5x}{3}-\frac{2}{3}-\frac{7x}{5}+\frac{3}{5}$

⇒ $\frac{15x-50x+42x}{30}\geqslant\frac{-10+9}{15}$

⇒ $\frac{7x}{30}\geqslant\frac{-1}{15}$

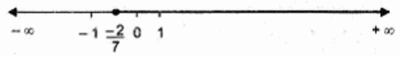
Multiplying both sides by 30, we have

$7x \geqslant -2$

Dividing both sides by 7, we have

The solution set is $\left[\frac{-2}{7}, \infty\right)$

The representation of the solution set on the number line is



47. Ravi obtained 70 and 75 marks in first two unit tests. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

Ans. : Let the marks obtained by Ravi in third test be x.

Then average of three tests $= \frac{70+75+x}{3}$

Now $\frac{70+75+x}{3} \geqslant 60 \Rightarrow \frac{145+x}{3} \geqslant 60$

Multiplying both sides by 3, we have

$145 + x \geqslant 180$

$\Rightarrow x \geqslant 180 - 145 \Rightarrow x \geqslant 35$

Thus the minimum marks needed to be obtained by Ravi = 35.

48. To receive Grade 'A', in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94. and 95, find minimum marks that Sunita must obtain in fifth examination to get Grade 'A' in the course.

Ans. : Let the marks obtained by Sunita in fifth examination be x.

Then average of five examinations $= \frac{87+92+94+95+x}{5}$

Now $\frac{87+92+94+95+x}{5} \geqslant 90 \Rightarrow \frac{368+x}{5} \geqslant 90$

Multiplying both sides by 5, we have

$368 + x \geqslant 450$

$\Rightarrow x \geqslant 450 - 368$

$\Rightarrow x \geqslant 82$

Thus the minimum marks needed to be obtained by Sunita = 82.

49. The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm. Find the minimum length of the shortest side.

Ans. : Let the length of the shortest side be x cm.

Then length of longest side = 3x cm

length of third side = (3x - 2)cm

Perimeter of triangle = x + 3x + 3x - 2

= (7x - 2)cm

Now $7x - 2 \geqslant 61$

$\Rightarrow 7x \geqslant 61 + 2 \Rightarrow 7x \geqslant 63 \Rightarrow x \geqslant 9$

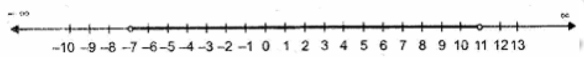
Thus the minimum length of shortest side = 9 cm

50. A man wants to cut three lengths from a single piece of board of length 91cm. The second length is to be 3cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5cm longer than the second? [Hint: If x is the length of the shortest board, then x , (x + 3) and 2x are the lengths of the second and third piece, respectively. Thus, x + (x + 3) + 2x $\leq$ 91 and 2x $\geq$ (x + 3) + 5].

Ans. : Let the length of the shortest board be x cm
Then length of the second board = (x + 3) cm
length of the third board = 2x cm
Now $x+(x+3)+2x \leqslant 91$ and $2x \geqslant (x+3)+5$
$\Rightarrow 4x+3 \leqslant 91$ and $2x-(x+3) \geqslant 5$
$\Rightarrow 4x \leqslant 91-3$ and $2x-x-3 \geqslant 5$
$\Rightarrow 4x \leqslant 88$ and $x \geqslant 5+3$
$\Rightarrow x \leqslant 22$ and $x \geqslant 8$
Thus minimum length of shortest board is 8 cm and maximum length is 22 cm.

51. Solve the inequality and represent the solution graphically on number line: 5 (2x – 7) – 3 (2x + 3) $\leq$ 0 , 2x + 19 $\leq$ 6x + 47.

Ans. : We have $5(2x-7)-3(2x+3) \leqslant 0$ and $2x+19 \leqslant 6x+47$
$\Rightarrow 10x-35-6x-9 \leqslant 0$ and $-4x \leqslant 28$
$\Rightarrow 4x-44 \leqslant 0$ and $x \geqslant -7$
$\Rightarrow 4x \leqslant 44$ and $x \geqslant -7$
$\Rightarrow x \leqslant 11$ and $x \geqslant -7$



52. A solution is to be kept between 68°F and 77°F. What is the range of temperature in degree Celsius (C) if the Celsius / Fahrenheit (F) convension formula is given by $F=\frac{9}{5}C+32$

Ans. : It is given that 68° < F < 77°
Putting $F=\frac{9}{5}C+32$, we have
$68^\circ < \frac{9}{5}C+32 < 77^\circ$
$\Rightarrow 36^\circ < \frac{9C}{5} < 45^\circ \Rightarrow 180^\circ < 9C < 225^\circ$
$\Rightarrow$ 20° < C < 25°
Thus the range of temperature between 20°C and 25°C

53. In an experiment, a solution of hydrochloric acid is to be kept between 30° and 35° Celsius. What is the range of temperature in degree Fahrenheit if conversion formula is given by C = $\frac{5}{9}$ (F – 32), where C and F represent a temperature in degree Celsius and degree Fahrenheit, respectively.

Ans. : It is given that 30 < C < 35.

Put, C = $\frac{5}{9}$ (F – 32), we get

30 < $\frac{5}{9}$ (F – 32) < 35,

$\Rightarrow$ $\frac{9}{5}$ $\times$ (30) < (F – 32) < $\frac{9}{5}$ $\times$ (35)

$\Rightarrow$ 54 + 32< (F – 32) + 32< 63 + 32

$\Rightarrow$ 86 < F < 95.

Thus, the required range of temperature is between 86° F and 95° F.

54. A company manufactures cassettes and its cost and revenue functions for a week are $\mathrm{C} = 300 + \frac{3}{2}\mathrm{x}$ R = 2x respectively, where x is the number of cassettes produced and sold in a week. How many cassettes must be sold for the company to realize a profit?

Ans. :

We have,

profit = Revenue - cost

therefore, to ear some profit, we must have

Revenue > Cost

$\Rightarrow 2\mathrm{x} > 300 + \frac{3}{2}\mathrm{x}$

$\Rightarrow 2\mathrm{x} - \frac{3}{2}\mathrm{x} > 300$

$\Rightarrow \frac{4\mathrm{x} - 3\mathrm{x}}{2} > 300$

$\Rightarrow \mathrm{x} > 300 \times 2$

$\Rightarrow \mathrm{x} > 600$

Hence, then manufacture must sell more that 600 cassettes to realize some profit.

55. A solution is to be kept between 86° and 95°F. What is the range of temperature in degree Celsius, if the Celsius (C)/ Fahrenheit (F) conversion formula is given by $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$.

Ans. :

We have,

F_1 = 86° F

$\therefore \mathrm{F}_1 = \frac{9}{5}\mathrm{C}_1 + 32 \quad \left[\because \mathrm{F} = \frac{9}{5}\mathrm{C} + 32\right]$

$\Rightarrow 86 = \frac{9}{5}\mathrm{C}_1 + 32$

$\Rightarrow 86 - 32 = \frac{9}{5}\mathrm{C}_1$

$\Rightarrow 54 = \frac{9}{5}\mathrm{C}_1$

$\Rightarrow 9\mathrm{C}_1 = 5 \times 54$

$\Rightarrow \mathrm{C}_1 = \frac{5 \times 54}{9}$

$\Rightarrow \mathrm{C}_1 = 5 \times 6 = 30^\circ\mathrm{C}$

Now, $\mathrm{F}_2 = 95^\circ\mathrm{F}$

$\therefore \mathrm{F}_2 = \frac{9}{5}\mathrm{C}_2 + 32$

$\Rightarrow 95-32=\frac{9}{5}C_2$
$\Rightarrow 63=\frac{9}{5}C_2$
$\Rightarrow 9C_2=63\times 5$
$\Rightarrow C_2=\frac{63\times 5}{9}$
$\Rightarrow C_2=7\times 5=35^{\circ}C$
The range of temperature of the solution is from 30ºC to 35ºC.

*** Given section consists of questions of 5 marks each.** **[20]**

56. A solution of 8% boric acid is to be diluted by adding a 2% boric acid solution to it. The resulting mixture is to be more than 4% but less than 6% boric acid. If we have 640 litres of the 8% solution, how many litres of the 2% solution will have to be added?

Ans. : Let x litre of 2% boric acid solution be added to 640 litres of 8% boric acid solution. Then
Total quality of mixture = (640 + x) litres
Total boric acid in (640 + x) litres of mixture $=\frac{2x}{100}+\frac{8}{100}\times 640$
$=\frac{x}{50}+\frac{256}{5}$
It is given that the resulting mixture must be more than 4% but less than 6% boric acid.
$\therefore \frac{4}{100}(640+x)<\frac{x}{50}+\frac{256}{5}<\frac{6}{100}(640+x)$
$\Rightarrow \frac{640+x}{25}<\frac{x+2560}{50}<\frac{1920+3x}{50}$
$\Rightarrow$ 1280 + 2x < x + 2560 < 1920 + 3x
$\Rightarrow$ 1280 + 2x < x + 2560 and x + 2560 < 1920 + 3x
$\Rightarrow$ x < 1280 and
$\Rightarrow$ x < 1280 and x > 320
$\Rightarrow$ 320 < x < 1280
Thus 2% boric acid solution must be more than 320 litres but less than 1280 litres.

57. How many litres of water will have to be added to 1125 litres of the 45% solution of acid so that the resulting mixture will contain more than 25% but less than 30% acid content?

Ans. : Let x litres of water be added to 1125 litres of 45% acid solution.
Then total quantity of mixture = (1125 +x) litres
$\frac{45}{100}\times 1125+0\times\frac{x}{100}>\frac{25}{100}\times(1125+x)$
and $\frac{45}{100}\times 1125+0\times\frac{x}{100}<\frac{30}{100}\times(1125+x)$
Combining the above inequations, we get
$\frac{25}{100}\times 100\leqslant\frac{2025\times 100}{4(1125+x)}\leqslant\frac{30}{100}\times 100$
$\Rightarrow 25\leqslant\frac{50625}{1125+x}\leqslant 30$
$\Rightarrow 25\leqslant\frac{50625}{1125+x}$ and $\frac{50625}{1125+x}\leqslant 30$

$\Rightarrow 28125+25x \leqslant 50625$ and $50625 \leqslant 33750+30x$

$\Rightarrow 25x \leqslant 22500$ and $30x \geqslant 1687.5$

$\Rightarrow x \leqslant 900$ and $x \geqslant 562.5$

$\Rightarrow 562.5 \leqslant x \leqslant 900$

Thus minimum 562.5 litres and maximum 900 litres of water need to be added.

58. A solution of 9% acid is to be diluted by adding 3% acid solution to it. The resulting mixture is to be more than 5% but less than 7% acid. If there is 460 litres of the 9% solution, how many litres of 3% solution will have to be added?

Ans. : Let Off 3% solution be added 460 L of 9% solution of acid.

Then, total quantity of mixture = (460 + x) L

Total acid content in the (460 + x) L of maxture $= \left(460 \times \frac{9}{100} + \mathrm{x} \times \frac{3}{100}\right)$

It is given that acid content in the resulting mixture must be more than 5% but less than 7% acid.

$\Rightarrow 5\%(460+\mathrm{x}) < 460 \times \frac{9}{100} + \frac{3\mathrm{x}}{100} < 7\%(460+\mathrm{x})$

$\Rightarrow \frac{5}{100} \times (460+\mathrm{x}) < 460 \times \frac{9}{100} + \frac{3\mathrm{x}}{100} < \frac{7}{100} \times (460+\mathrm{x})$

$\Rightarrow 5 \times (460+\mathrm{x}) < 460 \times 9 + 3\mathrm{x} < 7 \times (460+\mathrm{x})$

$\Rightarrow 2300+5\mathrm{x} < 4140+3\mathrm{x} < 3220+7\mathrm{x}$

$\Rightarrow 5\mathrm{x} < 1840+3\mathrm{x} < 920+7\mathrm{x}$

$\Rightarrow 2\mathrm{x} < 1840 < 920+4\mathrm{x}$

$\Rightarrow \mathrm{x} < 920+4\mathrm{x}$

$\Rightarrow \mathrm{x} < 920 < 230 < \mathrm{x}$

$\Rightarrow 230 < \mathrm{x} < 920$

Hence, the number of litres of the 3% solution of acid must be more than 230 and less than 920.

59. The water acidity in a pool is considerd normal when the average pH reading of three daily measurements is between 8.2 and 8.5. If the first two pH readings are 8.48 and 8.35, find the range of pH value for the third reading that will result in the acidity level being normal.

Ans. : Let the third pH value be x.

Given that first pH value = 8.48

Second pH value = 8.35

Average value of pH $= \frac{8.48+8.35+\mathrm{x}}{3}$

But average value pf pH lies batween 8.2 and 8.5

⇒ 24.6 < 16.83 + x < 25.5

⇒ 24.6 - 16.83 < x < 25.5 - 16.83

⇒ 7.77 < x < 8.67

Hence, the thrid pH value lies between 7.77 and 8.67.

*** Case study based questions** **[8]**

60. Shweta was teaching "method to solve a linear inequality in one variable" to her daughter.

Step I Collect all terms involving the variable (x) on one side and constant terms on other side with the help of above rules and then reduce it in the form $\mathbf{ax} < \mathbf{b}$ or $\mathbf{ax} \leq \mathbf{b}$ or $\mathbf{ax} > \mathbf{b}$ or $\mathbf{ax} \geq \mathbf{b}$.

Step II Divide this inequality by the coefficient of variable (x). This gives the solution set of given inequality.

Step III Write the solution set.

Based on above information, answer the following questions.

(i) The solution set of $\mathbf{24x} < \mathbf{100}$, when x is a natural number is

(a) $\{1,2,3,4\}$ (b) $(1,4)$ (c) $[1,4]$ (d) None of these

(ii) The solution set of $24100\mathrm{x} <$, when x is an integer is

(a) $\{\ldots\ldots -4,-3,-2,-1,0,1,2,3,4\}$ (b) $(-\infty,4]$ (c) $[4,\infty]$ (d) None of the above

(iii) The solution set of $-\mathbf{5x} + \mathbf{25} > 0$ is

(a) $[5,\infty)$ (b) $(-\infty,5]$ (c) $(5,\infty)$ (d) $(-\infty,5)$

(iv) The solution set of $\mathbf{3x} - \mathbf{5} < \mathbf{x} + \mathbf{7}$ is

(a) $(6,\infty)$ (b) $[6,\infty)$ (c) $(-\infty,6)$ (d) $(-\infty,6]$

(v) The solution set of $x + \frac{x}{2} + \frac{x}{3} < \mathbf{11}$ is

(a) $(-\infty,6]$ (b) $(-\infty,6)$ (c) $[6,\infty)$ (d) None of these

Ans. : (i) (a) We have, $\mathbf{24x} < \mathbf{100}$

On dividing both sides by 24 , we get

$= \frac{24x}{24} < \frac{100}{24} \Rightarrow x < \frac{25}{6}$

When $\mathbf{x}$ is a natural number, then solutions of the inequality $x < \frac{25}{6}$, are all natural numbers, which are less than $\frac{25}{6}$. In this case, the following values of x make the statement true.

$\mathrm{x} = 1,2,3,4$

Hence, the solution set of inequality is $\{\mathbf{1,2,3,4}\}$.

(ii) (a) We have, $24\mathrm{x} < \mathbf{100}$

On dividing both sides by 24 , we get

$= \frac{24x}{24} < \frac{100}{24} \Rightarrow x < \frac{25}{6}$

When $\mathbf{x}$ is an integer.

In this case, solutions of given inequality are

$\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4$

Hence, the solution set of inequality is

$\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4\}$

(iii) (d) We have, $-5x + 25 > 0$

On adding $5\mathbf{x}$ both sides, we get

$-5x + 25 + 5x > 0 + 5x \Rightarrow 25 > 5x \Rightarrow 5x < 25$

On dividing both sides by 5 , we get

$\Rightarrow \frac{5x}{5} < \frac{25}{5} \Rightarrow x < 5$

Hence, the required solution set is $(-\infty, 5)$.

(iv)

(c) We have, $3\mathbf{x} - \mathbf{5} < \mathbf{x} + \mathbf{7}$

$\Rightarrow 3x - 5 + 5 < x + 7 + 5$

$\Rightarrow 3\mathrm{x} < \mathrm{x} + 12$

$\Rightarrow 3\mathrm{x} - \mathrm{x} < \mathrm{x} + 12 - \mathrm{x}$

[adding 5 both sides]

[subtracting $\mathbf{x}$ from both sides]

$$\begin{aligned} \Rightarrow \quad & 2x < 12 \\ \Rightarrow \quad & \frac{2x}{2} < \frac{12}{2} \\ \Rightarrow \quad & x < 6 \end{aligned}$$

[dividing both sides by 2]

The solution set is $\{\mathbf{x} : \mathbf{x} \in \mathbf{R}$ and $\mathbf{x} < 6\}$ i.e., any real number less than 6 . This can also be written as $(-\infty, 6)$.

(v) (b) We have, $x + \frac{x}{2} + \frac{x}{3} < 11$

$\Rightarrow \frac{6x+3x+2x}{6} < 11 \Rightarrow \frac{11x}{6} < 11$

On multiplying both sides by $\frac{6}{11}$, we get

$$\frac{11x}{6} \times \frac{6}{11} < 11 \times \frac{6}{11} \Rightarrow x < 6$$

$$\therefore \quad x \in (-\infty, 6)$$

61. A manufacturing company produces certain goods. The company manager used to make a data record on daily basis about the cost and revenue of these goods separately. The cost and revenue function of a product are given by $C(x) = 20x + 4000$ and $R(x) = 60x + 2000$, respectively, where x is the number of goods produced and sold.

Based on above information, answer the following questions.

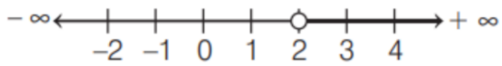
(i) How many goods must be sold to realise some profit?
(a) $\mathrm{x} < \mathbf{50}$
(b) $x > 50$
(c) $x \geq 50$
(d) $\mathbf{x} \leq \mathbf{50}$

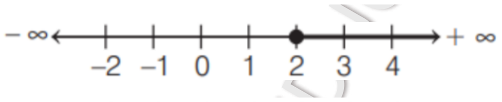
(ii) If the cost and revenue functions of a product are given by $C(x) = 3x + 400$ and $R(x) = 5x + 20$ respectively, where x is the number of items produced by the manufacturer, then how many items must be sold to realise some profit?
(a) $x \leq 190$
(b) $x \geq 190$
(c) $x < 190$
(d) $x > 190$

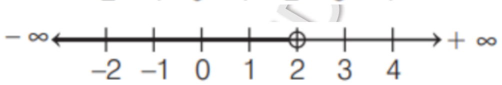
(iii) Let $\mathbf{x}$ and $\mathbf{b}$ are real numbers. If $\mathbf{b} > \mathbf{0}$ and $\mathbf{x} < \mathbf{b}$, then
(a) x is always positive
(b) $\mathbf{X}$ is always negative
(c) x is real number
(d) None of these

(iv) The solution set of $\mathbf{3} - \mathbf{5} < \mathrm{x} + \mathbf{7}$, when x is a whole number is given by
(a) $\{0, 1, 2, 3, 4, 5\}$
(b) $(-\infty, 6)$
(c) $[0, 5]$
(d) None of these

(v) Graph of inequality $x > 2$ on the number line is represented by

(a) 

(b) 

(c) 

(d) None of the above

Ans. : (i) (b) We know that, Profit = Revenue - Cost
$\therefore$ In order to realise some profit, revenue should be greater than the cost.
Thus, we should have $\mathbf{R}(\mathbf{x}) > \mathbf{C}(\mathbf{x})$

$$\Rightarrow 60x + 2000 \quad > 20x + 4000$$
$$\Rightarrow \quad 60x + 2000 - 20x > 20x + 4000 - 20x$$
$$[\text{ subtracting } 20x \text{ from both sides }]$$
$$\Rightarrow \quad 40x + 2000 > 4000$$
$$\Rightarrow \quad 40x + 2000 - 2000 > 4000 - 2000$$
$$\Rightarrow \quad [\text{ subtracting } 2000 \text{ from both sides }]$$
$$\Rightarrow 40x > 2000$$
$$\Rightarrow \frac{40x}{40} > \frac{2000}{40}$$
$$\Rightarrow x > 50$$

[subtracting 2000 from both sides]

[dividing both sides by 40]

$\Rightarrow x > 50$

Hence, the manufacturer must sell more than 50 items to realise some profit.

(iii) (d) We have, $\mathbf{b} > \mathbf{0}$

and $\mathbf{x} < \mathbf{b}$

Its mean $\mathbf{x}$ is always less than some positive quantity.

$\therefore x$ may be a real number.

(iv) (a) We have, $\mathbf{3x} - \mathbf{5} < \mathbf{x} + \mathbf{7}$

$$\Rightarrow 3x - 5 + 5 < x + 7 + 5 \quad [\text{adding } 5 \text{ both sides}]$$
$$\Rightarrow \quad 3x < x + 12$$
$$\Rightarrow \quad 3x - x < x + 12 - x$$

[adding 5 both sides]

[subtracting $\mathbf{x}$ from both sides]

$$\Rightarrow 2x < 12$$
$$\Rightarrow \frac{2x}{2} < \frac{12}{2}$$

[dividing both sides by 2]

$\Rightarrow x < 6$

Now, if $\mathbf{x}$ is a whole number, then the solution set $\{0,1,2,3,4,5\}$.

(v) (b)

----- "If you are working on something that you really care about, you don't have to be pushed. The vision pulls you. -----